Factored Form of a Quadratic

Quadratics may appear in many different forms:

1. Standard Form... $y = ax^2 + bx + c$

2. Factored Form... $y = a(x-r)(x-s)$

3. Vertex Form... $y = a(x-p)^2 + q$

We're going to examine Factored Form today.

INVESTIGATE the Math

Ataneq takes tourists on dogsled rides. He needs to build a kennel to separate some of his dogs from the other dogs in his team. He has budgeted for 40 m of fence. He plans to place the kennel against part of his home, to save on materials.

? What dimensions should Ataneq use to maximize the area of the kennel?

A. Using x to represent the width of the kennel, create an expression for the length of the kennel.

B. Write a function, in terms of x, that defines the area of the kennel. Identify the factors in your function.

C. Create a table of values for the function, and then graph it.

D. Does the function contain a maximum or a minimum value? Explain.

E. Determine the x-intercepts of the parabola.

F. Determine the equation of the axis of symmetry of the parabola and the coordinates of the vertex.

G. What are the dimensions that maximize the area of the kennel?

Reflecting

H. How are the x-intercepts of the parabola related to the factors of your function?

I. Explain why having a quadratic function in factored form is useful when graphing the parabola.

EXAMPLE 1 Graphing a quadratic function given in standard form

Sketch the graph of the quadratic function:

$$f(x) = 2x^2 + 14x + 12$$

State the domain and range of the function.

Your turn....

Sketch the graph of the following

$f(x) = -3x^2 + 6x - 3$

EXAMPLE 2 Using a partial factoring strategy to sketch the graph of a quadratic function

Sketch the graph of the following quadratic function:

$$f(x) = -x^2 + 6x + 10$$

State the domain and range of the function.

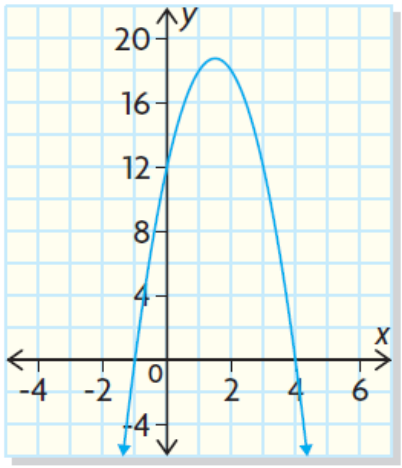
Your turn...

Apply the partial factoring strategy to locate two points that have the same y-coordinate on the following function.

$f(x) = -x^2 - 3x + 12$

EXAMPLE 3 Determining the equation of a quadratic function

Determine the function that defines this parabola. Write the function in standard form.



Key Ideas

- When a quadratic function is written in factored form

$$y = a(x - r)(x - s)$$

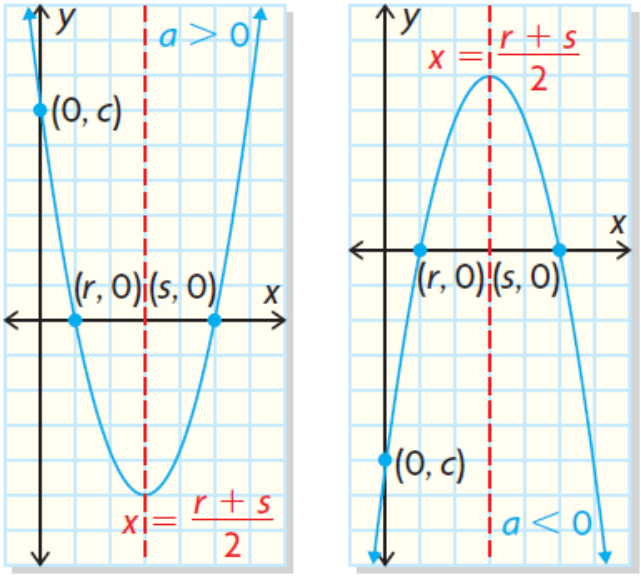
each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.
- If a parabola has one or two x-intercepts, the equation of the parabola can be written in factored form using the x-intercept(s) and the coordinates of one other point on the parabola.
- Quadratic functions without any zeros cannot be written in factored form.

Need to Know

- A quadratic function that is written in the form

$$f(x) = a(x - r)(x - s)$$

has the following characteristics:
  - The x-intercepts of the graph of the function are $x = r$ and $x = s$.
  - The linear equation of the axis of symmetry is $x = \frac{r + s}{2}$.
  - The y-intercept, c, is $c = a \cdot r \cdot s$.



- If a quadratic function has only one x-intercept, the factored form can be written as follows:

$$f(x) = a(x - r)(x - r)$$

$$f(x) = a(x - r)^2$$