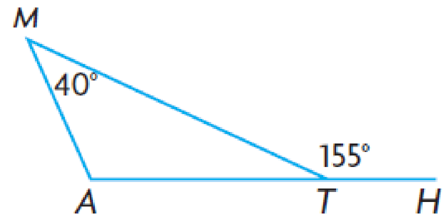
ANGLE PROPERTIES IN
Triangles and Polygons

Goal: To prove properties of angles in triangles and use those properties to solve problems.

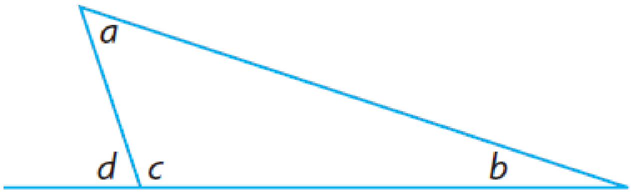
EXAMPLE 1 Using angle sums to determine angle measures

In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



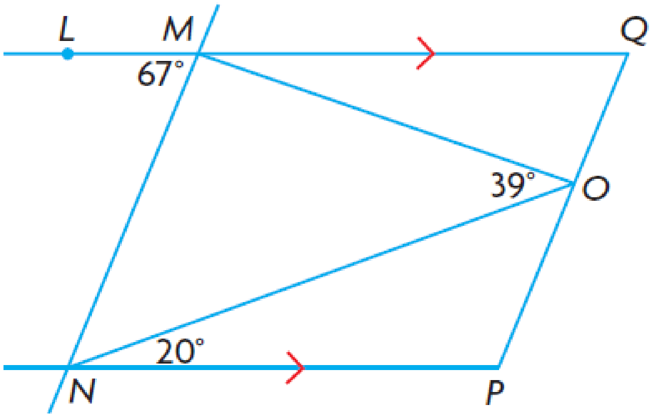
EXAMPLE 2 Using reasoning to determine the relationship between the exterior and interior angles of a triangle

Determine the relationship between an exterior angle of a triangle and its **non-adjacent interior angles**.



EXAMPLE 3 Using reasoning to solve problems

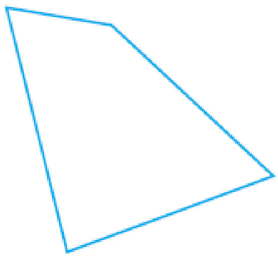
Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



Angle Properties In Polygons

Goal: To prove properties of angles in polygons and use those properties to solve problems.

Part 1 Interior Angles



A. Giuseppe says that he can determine the sum of the measures of the interior angles of this quadrilateral by including the diagonals in the diagram. Is he correct? Explain.

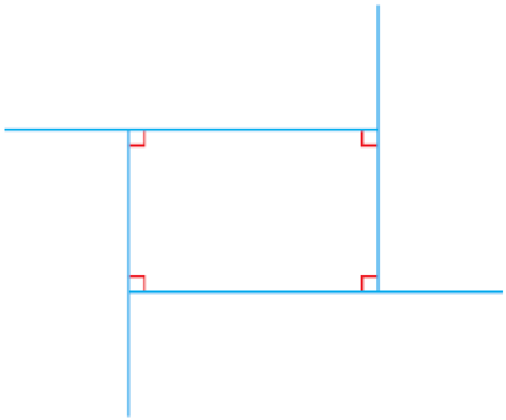
B. Determine the sum of the measures of the interior angles of any quadrilateral.

C. Draw the polygons listed in the table below. Create triangles to help you determine the sum of the measures of their interior angles. Record your results in a table like the one below.

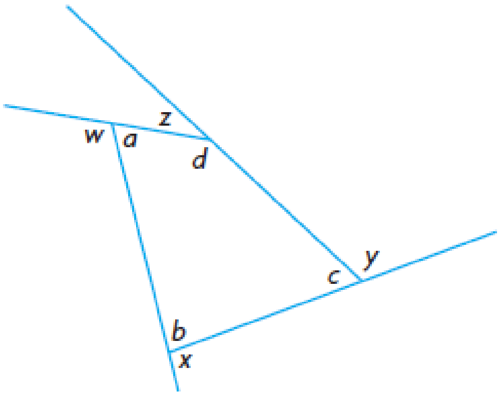
Polygon	Number of Sides	Number of Triangles	Sum of Angle Measures
triangle	3	1	180°
quadrilateral	4		
pentagon	5		
hexagon	6		
heptagon	7		
octagon	8		

Part 2 Exterior Angles

F. Draw a rectangle. Extend each side of the rectangle so that the rectangle has one exterior angle for each interior angle. Determine the sum of the measures of the exterior angles.



G. What do you notice about the sum of the measures of each exterior angle of your rectangle and its adjacent interior angle? Would this relationship also hold for the exterior and interior angles of the irregular quadrilateral shown? Explain.



H. Make a conjecture about the sum of the measures of the exterior angles of any quadrilateral. Test your conjecture.

I. Draw a pentagon. Extend each side of the pentagon so that the pentagon has one exterior angle for each interior angle. Based on your diagram, revise your conjecture to include pentagons. Test your revised conjecture.

J. Do you think your revised conjecture will hold for polygons that have more than five sides? Explain and verify by testing.

Reflecting

K. Compare your results for the sums of the measures of the interior angles of polygons with your classmates' results. Do you think your conjecture from part *D* will be true for any polygon? Explain.

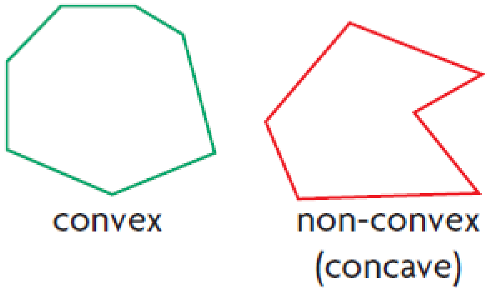
L. Compare your results for the sums of the measures of the exterior angles of polygons with your classmates' results. Do you think your conjecture from part I will apply to any polygon? Explain.

EXAMPLE 1 Reasoning about the sum of the interior angles of a polygon

Prove that the sum of the measures of the interior angles of any n-sided **convex polygon** can be expressed as $180°(n - 2)$.

convex polygon

A polygon in which each interior angle measures less than 180°.



EXAMPLE 2 Reasoning about angles in a regular polygon

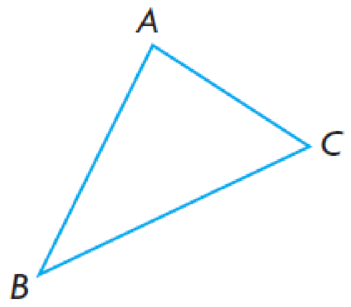
Outdoor furniture and structures like gazebos sometimes use a regular hexagon in their building plan. Determine the measure of each interior angle of a regular hexagon.

EXAMPLE 3 Visualizing tessellations

A floor tiler designs custom floors using tiles in the shape of regular polygons. Can the tiler use congruent regular octagons and congruent squares to tile a floor, if they have the same side length?

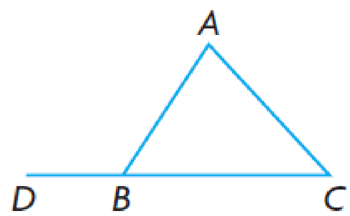
Need to Know

- In any triangle, the sum of the measures of the interior angles is proven to be 180°.



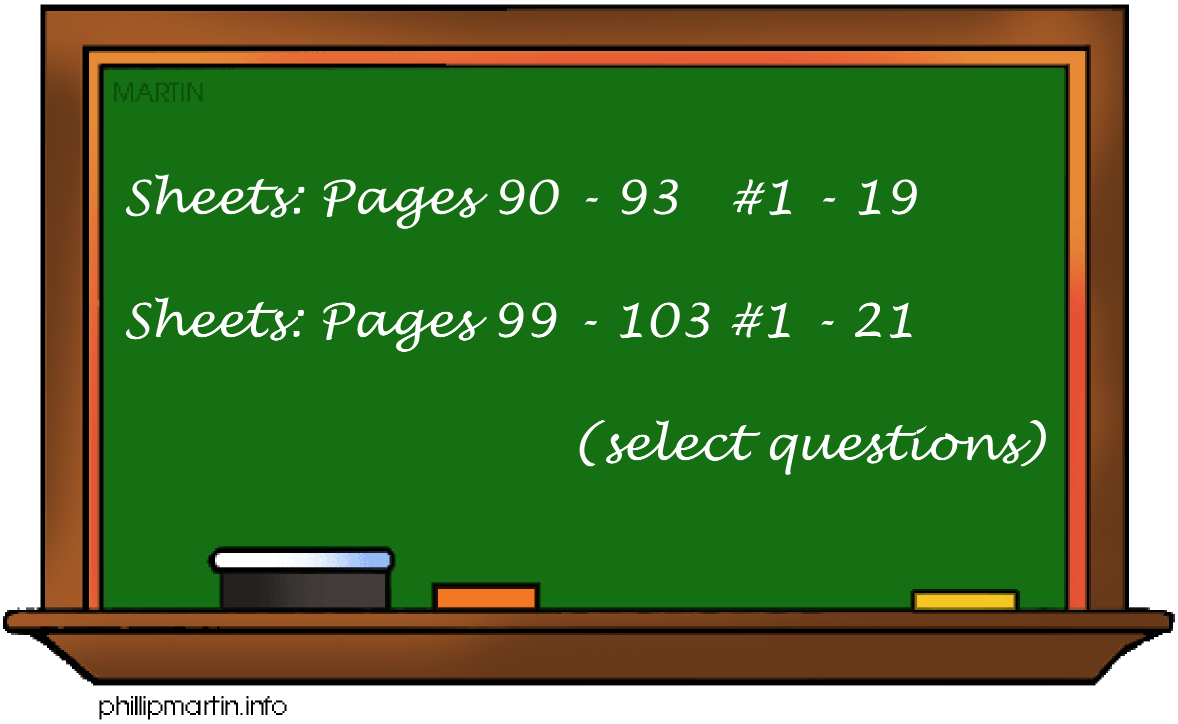
$\angle A + \angle B + \angle C = 180°$

- The measure of any exterior angle of a triangle is proven to be equal to the sum of the measures of the two non-adjacent interior angles.



$\angle DBA = \angle BAC + \angle ACB$

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180°(n - 2)$.
- The measure of each interior angle of a regular polygon is $\dfrac{180°(n - 2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360°.


MARTIN
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(select questions)
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