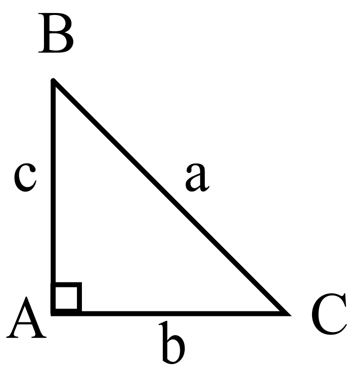
PROVING AND APPLYING the Law of Sines

Sketch a right angle triangle and label its vertices and sides.

Record the sine ratio for two of the angles.

What do you notice?



$$\text{Sin } B = \frac{b}{a} \qquad \text{Sin } C = \frac{c}{a}$$

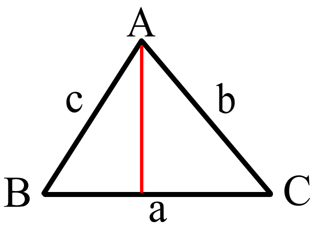
$$a\text{Sin}B = b \qquad a\text{Sin}C = c$$

$$a = \frac{b}{\text{Sin}B} \qquad a = \frac{c}{\text{Sin}C}$$

Transitive Property

Therefore, $\frac{b}{\text{Sin}B} = \frac{c}{\text{Sin}C}$

What if we try an acute triangle?
Will we still see this relationship?



Since we can't use basic trig here, let's drop an altitude to create 2 right angle triangles.

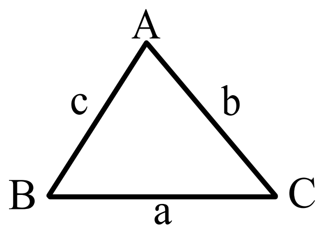
$\text{Sin}B = \frac{h}{c}$ | $\text{Sin}C = \frac{h}{b}$

$c\text{Sin}B = h$ | $b\text{Sin}C = h$

$\therefore c\text{Sin}B = b\text{Sin}C$

$$\frac{\text{Sin}B}{b} = \frac{\text{Sin}C}{c}$$

What about the other angle though?
Let's draw another altitude
and try...



$\text{Sin}A = \frac{h}{b}$ | $\text{Sin}B = \frac{h}{a}$

$b\text{Sin}A = h$ | $a\text{Sin}B = h$

$$b\text{Sin}A = a\text{Sin}B$$

$$\frac{\text{Sin}A}{a} = \frac{\text{Sin}B}{b}$$

So, we discover the Sine Law:

Since they are ratios, they can be flipped to read:

In Summary

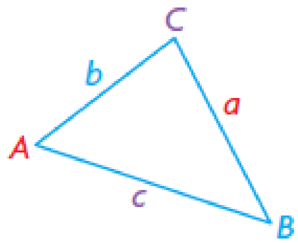
Key Idea

- The ratios of $\frac{\text{length of opposite side}}{\sin(\text{angle})}$ are equivalent for all three side–angle pairs in an acute triangle.

Need to Know

- In an acute triangle, $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



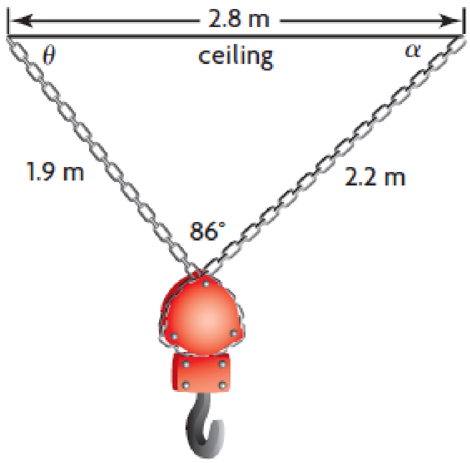
EXAMPLE 1 Using reasoning to determine the length of a side

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.

Proving Applying Sine Law

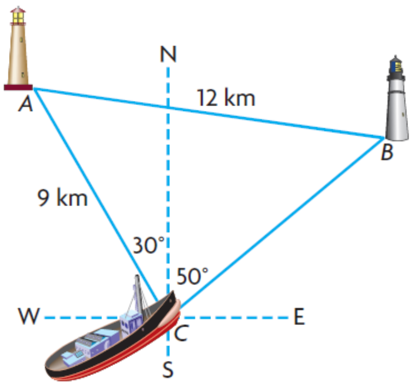
EXAMPLE 2 Solving a problem using the sine law

Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.



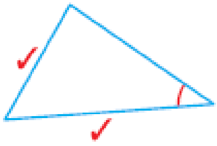
EXAMPLE 3 Using reasoning to determine the measure of an angle

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. Determine the compass direction he must follow when he leaves lighthouse *B* for lighthouse *A*.

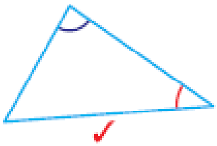


Need to Know

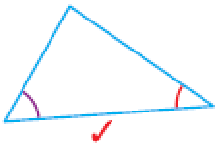
- You can use the sine law to solve a problem modelled by an acute triangle when you know:
  - two sides and the angle opposite a known side.



  - two angles and any side.



or



- If you know the measures of two angles in a triangle, you can determine the third angle because the angles must add to 180°.

- When determining side lengths, it is more convenient to use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- When determining angles, it is more convenient to use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Assignment:

Sheets Page 117 #1a, 3, 4

Sheets Page 124 #1-5, 7, 10
13 - 15, 17

** Review page 129 #1-9