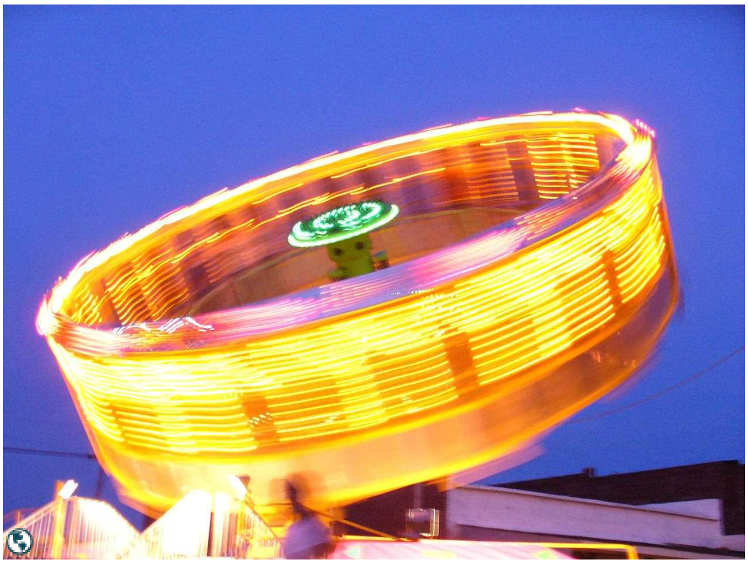
Periodic Motion

When an object repeats its motion it is considered to give periodic motion.

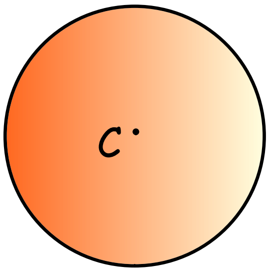
Circular Motion

Our bodies cannot sense constant velocity, but it is a sensitive accelerometer. It can detect acceleration in an elevator or plane departures/landings.

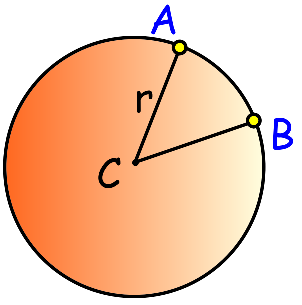
In carnival rides, even though the ride is moving at a constant speed, your direction is constantly changing.



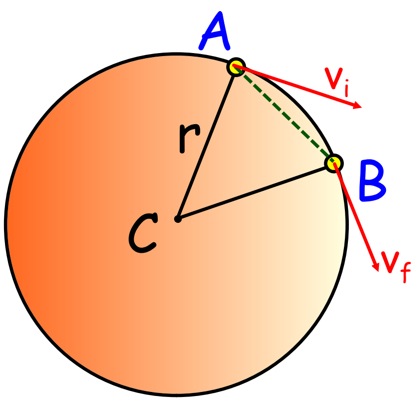
If we picture this event from a top-down perspective we see a circle.



If we picture a person spinning during the ride then we could isolate two points (A and B) moving with uniform circular motion.



The radius and speed are constant so the velocity is always perpendicular to the radius (tangent to the circle).



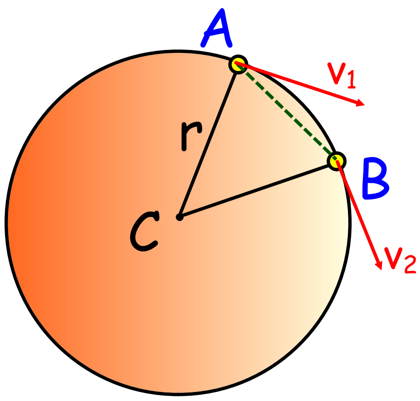
v_i = the instantaneous velocity at A

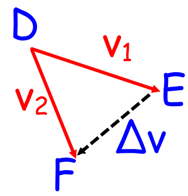
v_f = the instantaneous velocity at B

Accleration is a change in velocity divided by the time interval.

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ [have not seen vector subtraction yet]

$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$





Using similar triangles ABC and DEF we can create a ratio

$$\frac{\Delta\vec{v}}{\vec{v}} = \frac{AB}{r}$$

Because the ride is in uniform motion, the distance from A to B would have a constant velocity. Therefor $d = \vec{v}\Delta t$

$$\frac{\Delta\vec{v}}{\vec{v}} = \frac{\vec{v}\Delta t}{r}$$

Since acceleration is given by

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \text{then} \qquad \frac{\Delta\vec{v}}{\vec{v}} = \frac{\vec{v}\Delta t}{r}$$

$$a = \frac{\vec{v}^2}{r}$$

Newton originated the word "centripital" to indicate this change in velocity.

Centripital Acceleration equation

$$\vec{a}_c = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}^2}{r}$$

centripital acceleration *always* points toward the center of the circle.

It is often difficult to measure the speed of an object, but it is easier to measure how long it takes to make one complete revolution, or **period**, **T**.

During this time, the object travels around the circumference of the circle

The speed of the object is then:

$$v = \frac{d}{t} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T}$$

Substituting this into our centripital acceleration equation we get

$$a = \frac{\vec{v}^2}{r} =$$

Newton's second law tells us that an object does not accelerate unless a net force acts on it.

In the case of circular motion, these are called **centripital forces**, and are directed toward the center of the circle.

$$F_{net} = F_c = ma_c =$$

Example

A 0.013 kg rubber stopper is attached to a 0.93 m length of string. The stopper is swung in a horizontal circle, making one revolution in 1.18 seconds.

a) Find the speed of the stopper.
b) Find its centripetal acceleration
c) Find the force the string exerts on it.