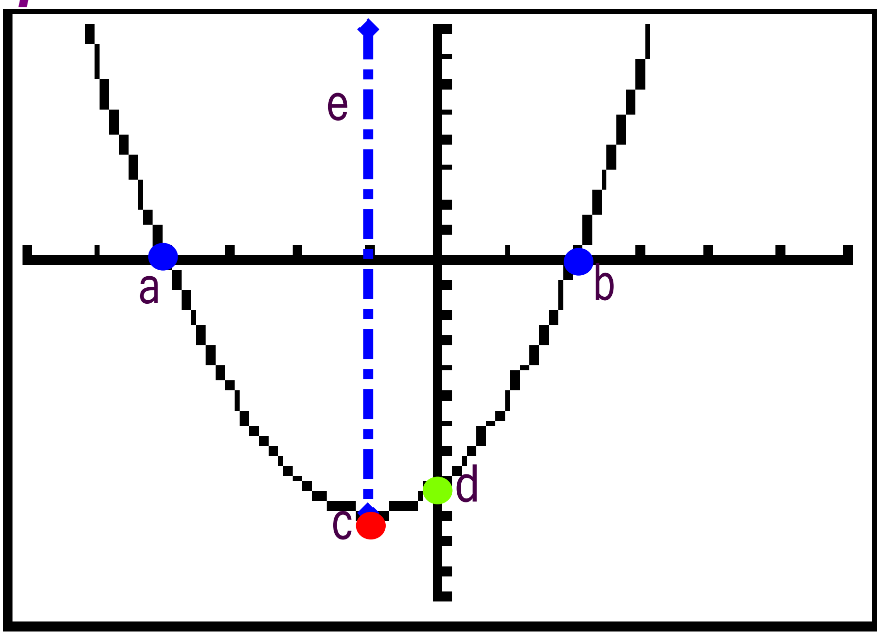
INVESTIGATING STANDARD FORM

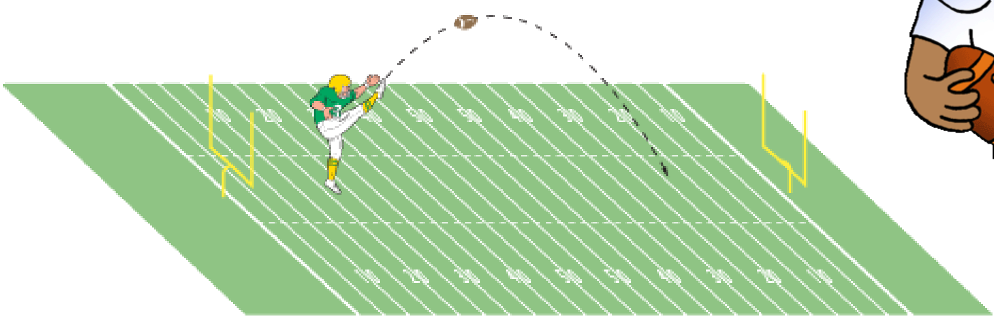
$$y = ax^2 + bx + c$$

- We can graph quadratics in this form by finding the important elements of a parabola like the ***roots, axis of symmetry, vertex, and y-intercept***.



INVESTIGATING QUADRATIC FUNCTIONS IN STANDARD FORM

When a player kicks or punts a football into the air, it reaches a maximum height before falling back to the ground. The moment it leaves the punter's foot to the moment it is caught or hits the ground is called the *hang time* of the punt. A punter attempts to kick the football so there is a longer hang time to allow teammates to run downfield to tackle an opponent who catches the ball. The punter may think about exactly where or how far downfield the football will land. How can you mathematically model the path of a football through the air after it is punted?



Standard Form of Quadratic

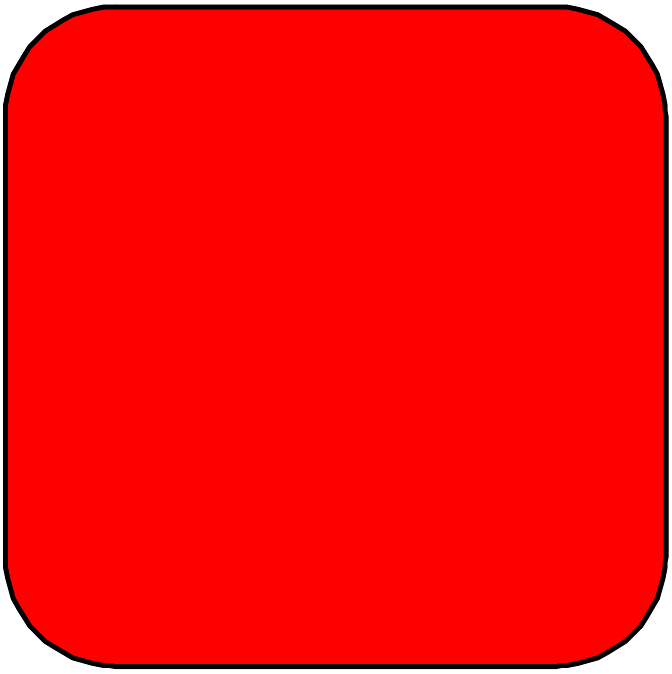
Investigate a Quadratic Function of the Form $f(x) = ax^2 + bx + c$

The path of a football through the air is just one of many real-life phenomena that can be represented by a quadratic function. A quadratic function of the form $f(x) = ax^2 + bx + c$ is written in **standard form**.

9. Using technology, graph the quadratic function $f(x) = -x^2 + 4x + 5$.

10. Describe any symmetry that the graph has.

11. Does the function have a maximum y-value? Does it have a minimum y-value? Explain.



12. Using technology, graph on a Cartesian plane the functions that result from substituting the following c-values into the function $f(x) = -x^2 + 4x + c$.

10

0

−5

13. Using technology, graph on a Cartesian plane the functions that result from substituting the following a-values into the function $f(x) = ax^2 + 4x + 5$.

−4

−2

1

2

14. Using technology, graph on a Cartesian plane the functions that result from substituting the following b-values into the function $f(x) = -x^2 + bx + 5$.

2

0

−2

−4

Standard Form of Quadratic

Reflect and Respond

15. What do your graphs show about how the function $f(x) = ax^2 + bx + c$ is affected by changing the parameter c?
16. How is the function affected when the value of a is changed? How is the graph different when a is a positive number?
17. What effect does changing the value of b have on the graph of the function?

Do any of the parameters affect the *position* of the graph?

Do any affect the *shape* of the graph?

STANDARD FORM

$$y = ax^2 + bx + c$$

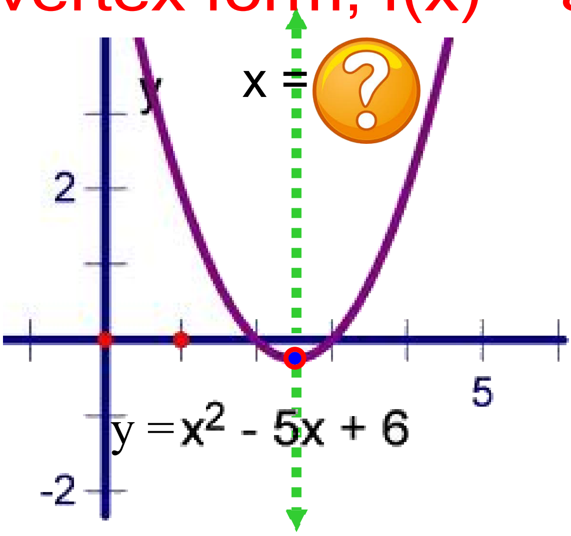
What we should know now...

- a determines the shape and whether the graph opens up ($a>0$) or downward ($a<0$).
- b influences the position of the graph.
- c determine the y-intercept of the graph.

You can expand vertex form, $f(x) = a(x - p)^2 + q$, and compare the resulting coefficients with the standard form, $f(x) = ax^2 + bx + c$, to see the relationship between the parameters of the two forms of a quadratic function.

Here goes: $f(x) = a(x - p)^2 + q$

Remember, to determine the x-coordinate of the vertex, you can use the equation x = p from vertex form, $f(x) = a\,(x - p)^2 + q$.



$$x = -\frac{b}{2a}$$

So, the x-coordinate of the vertex from standard form, $y = ax^2 + bx + c$, is $x = -b/2a$. This is also the axis of symmetry.

Standard Form of Quadratic

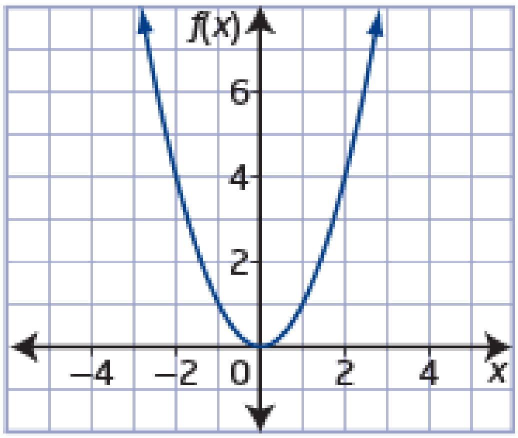
Example 1

Identify Characteristics of a Quadratic Function in Standard Form

For each graph of a quadratic function, identify the following:

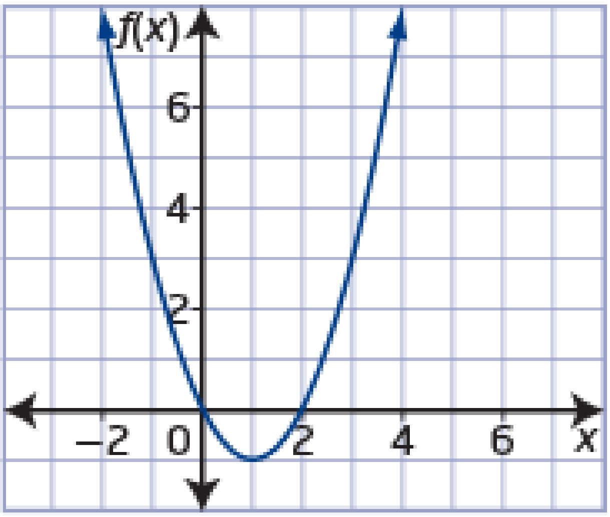
- the direction of opening
- the coordinates of the vertex
- the maximum or minimum value
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the domain and range

a) $f(x) = x^2$



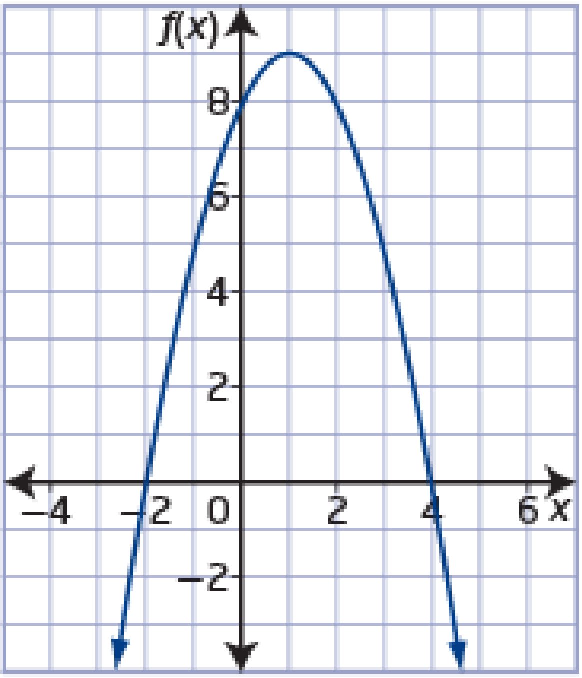
Standard Form of Quadratic

b) $f(x) = x^2 - 2x$



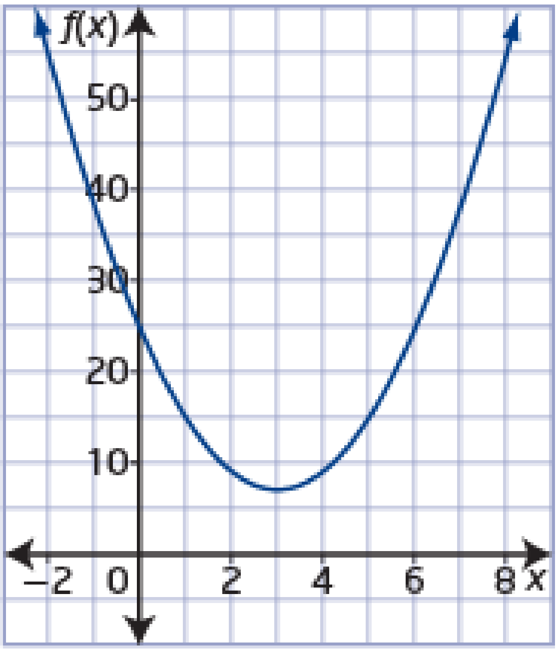
Standard Form of Quadratic

c) $f(x) = -x^2 + 2x + 8$



Standard Form of Quadratic

d) $f(x) = 2x^2 - 12x + 25$



EXTRA PRACTICE

For each quadratic function, identify the following:

- the direction of opening
- the coordinates of the vertex
- the maximum or minimum value
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the domain and range

a) $y = x^2 + 6x + 5$

b) $y = -x^2 + 2x + 3$

PROBLEM...

Pull

A frog sitting on a rock jumps into a pond. The height, h, in centimetres, of the frog above the surface of the water as a function of time, t, in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 150t + 25$. Where appropriate, answer the following questions to the nearest tenth.

a) Sketch the function, labelling as many important aspects as possible.

b) What is the y-intercept?

What does it represent in this situation?

c) What maximum height does the frog reach?

When does it reach that height?

d) When does the frog hit the surface of the water?

e) What are the domain and range in this situation?

f) How high is the frog 0.25 seconds after it jumps?

Pull

YOU TRY...

A diver jumps from a 3-m springboard with an initial vertical velocity of 6.8 m/s. Her height, h, in metres, above the water t seconds after leaving the diving board can be modelled by the function $h(t) = -4.9t^2 + 6.8t + 3$.

a) Sketch the function.
b) What does the y-intercept represent?
c) What maximum height does the diver reach? When does she reach that height?
d) How long does it take before the diver hits the water?
e) What domain and range are appropriate in this situation?
f) What is the height of the diver 0.6 seconds after leaving the board?

Write a Quadratic Function to Model a Situation

A rancher has 100 metres of fencing available to build a rectangular corral.

a) Write a quadratic function in standard form to represent the area of the corral.

b) What are the coordinates of the vertex? What does the vertex represent in this situation?

c) Sketch the graph for the function you determined in part a).

d) Determine the domain and range for this situation.

e) Identify any assumptions you made in modelling this situation mathematically.