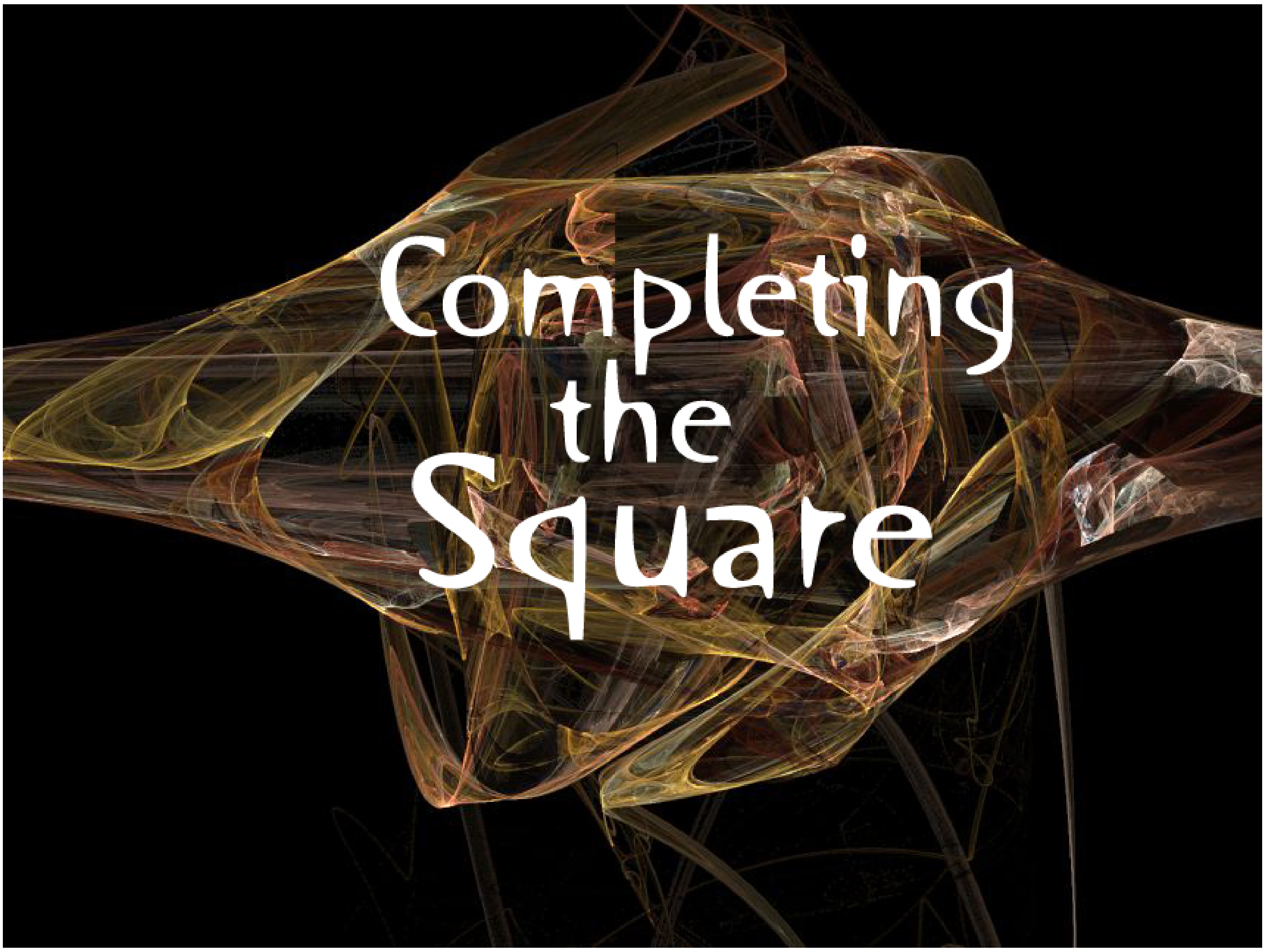
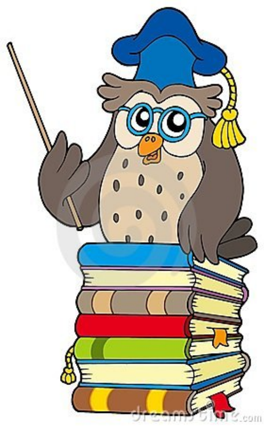
**Completing The Square**



We know quadratic functions can be expressed in:

- standard form $f(x) = ax^2 + bx + c$
- vertex form $f(x) = a(x - p)^2 + q$.

We can determine the direction of opening and general shape from both forms, but vertex form has the advantage that we can identify the co-ordinates of the vertex right away.



Since we're not always given quadratic functions in vertex form and since it is such a useful form, it is wise to be able to convert a function from standard from to vertex form.

We can convert from standard form to vertex form using an algebraic process called:
"Completing the Square".

This process involves adding a value, then subtracting that value from a quadratic polynomial so that it contains a perfect square trinomial. We can then rewrite this trinomial as the square of a binomial.

**Not making sense? Let's look at a few...**

## Example 1:

$y = x^2 - 8x + 5$


Group the first two terms.

Add and subtract the square of half the coefficient of the *x*-term.

Group the perfect square trinomial.

Rewrite as the square of a binomial.

Simplify.

Both the standard form, $y = x^2 - 8x + 5$ and the vertex form, $y = (x-4)^2 - 11$ represent the same quadratic function!

We know using both forms that the graph opens up and that a=1, but it is only in vertex form that we see the vertex is at (4, -11).

## Example 2:

Rewrite each function in vertex form by completing the square.

**a)** $f(x) = x^2 + 6x + 5$

**b)** $f(x) = 3x^2 - 12x - 9$

**c)** $f(x) = -5x^2 - 70x$

## Example 3:

### Convert to Vertex Form and Verify

**a)** Convert the function $y = 4x^2 - 28x - 23$ to vertex form.

**b)** Verify that the two forms are equivalent.

## Example 4:

### Determine the Vertex of a Quadratic Function by Completing the Square

Consider the function $y = 5x^2 + 30x + 41$.

**a)** Complete the square to determine the vertex and the maximum or minimum value of the function.

**b)** Use the process of completing the square to verify the relationship between the value of $p$ in vertex form and the values of $a$ and $b$ in standard form.

**c)** Use the relationship from part b) to determine the vertex of the function. Compare with your answer from part a).

**a)** $y = 5x^2 + 30x + 41$

$y = 5(x^2 + 6x) + 41$

$y = 5(x^2 + 6x + 9 - 9) + 41$

$y = 5[(x^2 + 6x + 9) - 9] + 41$

$y = 5[(x + 3)^2 - 9] + 41$

$y = 5(x + 3)^2 - 45 + 41$

$y = 5(x + 3)^2 - 4$

**b)**

Look back at the steps in completing the square.

$y = ax^2 + bx + 41$

$y = 5x^2 + 30x + 41$

$y = 5(x^2 + 6x) + 41$

$\vdots$

$y = 5(x + 3)^2 - 4$

$y = 5(x - p)^2 - 4$

$b$ divided by $a$ gives the coefficient of $x$ inside the brackets.

6 is $\frac{30}{5}$, or $\frac{b}{a}$.

Half the coefficient of $x$ inside the brackets gives the value of $p$ in the vertex form.

3 is half of 6, or half of $\frac{b}{a}$, or $\frac{b}{2a}$.

Considering the steps in completing the square, the value of $p$ in vertex form is equal to $-\frac{b}{2a}$. For any quadratic function in standard form, the equation of the axis of symmetry is $x = -\frac{b}{2a}$.

**c)** Determine the $x$-coordinate of the vertex using $x = -\frac{b}{2a}$.

$x = -\frac{30}{2(5)}$

$x = -\frac{30}{10}$

$x = -3$

Determine the $y$-coordinate by substituting the $x$-coordinate into the function.

$y = 5(-3)2 + 30(-3) + 41$

$y = 5(9) - 90 + 41$

$y = 45 - 90 + 41$

$y = -4$

The vertex is $(-3, -4)$.
This is the same as the coordinates for the vertex determined in part a).

**<u>Assignment:</u>**
Pre-Calculus Text Page 192 -
#2 - 4, 7(a,b,c), 8, 12, 14, 16, 25