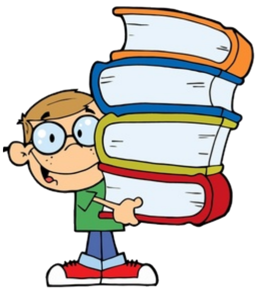
# CHAPTER 4
# Quadratic Equations

We will relate quadratic equations to the graphs of quadratic functions and solve problems by determining and analyzing quadratic equations.

**Quadratic equations are used in many situations.**



**Read page 204 & 205 now.**
**Copy the key terms on page 204 into your notes.**
**Copy definitions from page 208 into your notes.**


# WHAT IS A QUADRATIC EQUATION?

A quadratic equation is a second-degree equation with standard form, $ax^2 + bx + c = 0$ where $a \neq 0$.

Example: $2x^2 + 12x + 16 = 0$

# SOLVING A QUADRATIC EQUATION

## What does it mean to solve a quadratic equation?

"Solving" a quadratic equation means to determine the value of "x" that will make the function equal to zero. Therefore, the solutions to a quadratic equation are known as "zeros of a function" or "roots".

There is more than one way to solve a quadratic equation. Here are the ways we will explore:

1. By GRAPHING
2. By FACTORING
3. By COMPLETING THE SQUARE
4. Using THE QUADRATIC FORMULA

# SOLVE BY GRAPHING

* We can determine the solutions of a equation by determining the corresponding function and then graphing it.

* The solutions, roots of the equation, or the zeros of the function are visible as the x-intercepts when graphed.

* We can graph manually OR on the graphing calculator.

Example 1: Solve $x^2 - 2x - 8 = 0$ graphically.

Example 2: Determine the zeros of the function of $x^2 - 4x - 5 = 0$ by graphing.

# Solving Quadratic Equations by Graphing

Water fountains are usually designed to give a specific visual effect. For example, the water fountain shown consists of individual jets of water that each arch up in the shape of a parabola. Notice how the jets of water are designed to land precisely on the underwater spotlights.

How can you design a water fountain to do this? Where must you place the underwater lights so the jets of water land on them? What are some of the factors to consider when designing a water fountain? How do these factors affect the shape of the water fountain?

# Solving Quadratic Equations by Graphing

1. Each water fountain jet creates a parabolic stream of water. You can represent this curve by the quadratic function $h(x) = -6(x - 1)^2 + 6$, where $h$ is the height of the jet of water and $x$ is the horizontal distance of the jet of water from the nozzle, both in metres.

a) Graph the quadratic function $h(x)=-6(x-1)^2 + 6$

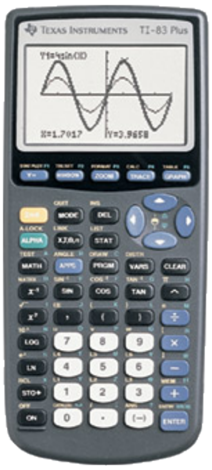
b) How far from the nozzle should the underwater lights be placed? Explain.

2. You can control the height and horizontal distance of the jet of water by changing the water pressure. Suppose that the quadratic function $h(x) = -x^2 + 12x$ models the path of a jet of water at maximum pressure. The quadratic function $h(x) = -3x^2 + 12x$ models the path of the same jet of water at a lower pressure.

   a) Graph these two functions on the same set of axes as in step 1.

   b) Describe what you notice about the $x$-intercepts and height of the two graphs compared to the graph in step 1.

   c) Why do you think the $x$-intercepts of the graph are called the zeros of the function?



## Reflect and Respond

3. a) If the water pressure in the fountain must remain constant, how else could you control the path of the jets of water?

   b) Could two jets of water at constant water pressure with different parabolic paths land on the same spot? Explain your reasoning.

**Did You Know?**

The Dubai Fountain at the Burj Khalifa in Dubai is the largest in the world. It can shoot about 22 000 gal of water about 500 ft into the air and features over 6600 lights and 25 colour projectors.

## Example 1

### Quadratic Equations With One Real Root

What are the roots of the equation $-x^2 + 8x - 16 = 0$?

## Example 2

### Quadratic Equations With Two Distinct Real Roots

The manager of Jasmine's Fine Fashions is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100 + 15x - x^2$ gives the store's revenue $R$, in dollars, from dress sales, where $x$ is the price change, in dollars. What price changes will result in no revenue?

## Example 3

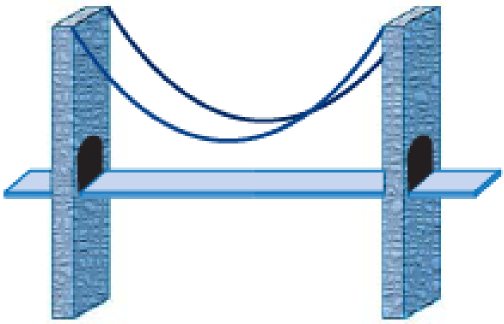
### Quadratic Equations With No Real Roots

Solve $2x^2 + x = -2$ by graphing.

## Example 4

### Solve a Problem Involving Quadratic Equations

The curve of a suspension bridge cable attached between the tops of two towers can be modelled by the function $h(d) = 0.0025(d - 100)^2 - 10$, where $h$ is the vertical distance from the top of a tower to the cable and $d$ is the horizontal distance from the left end of the bridge, both in metres. What is the horizontal distance between the two towers? Express your answer to the nearest tenth of a metre.



# YOUR TURN...

1. Determine the roots of the quadratic equation $x^2 - 6x + 9 = 0$.

2. The manager at Suzie's Fashion Store has determined that the function $R(x) = 600 - 6x^2$ models the expected weekly revenue, R, in dollars, from sweatshirts as the price changes, where x is the change in price, in dollars. What price increase or decrease will result in no revenue?

3. Solve $3m^2 - m = -2$ by graphing.

4. Suppose the cable of the suspension bridge in Example 4 is modelled by the function $h(d) = 0.0025(d - 100)^2 - 12$. What is the horizontal distance between the two towers? Express your answer to the nearest tenth of a metre.