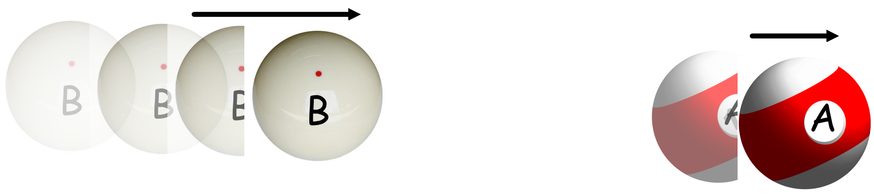
The Conservation of Momentum

To study momentum changes in collisions, we must use a **closed, isolated system**.

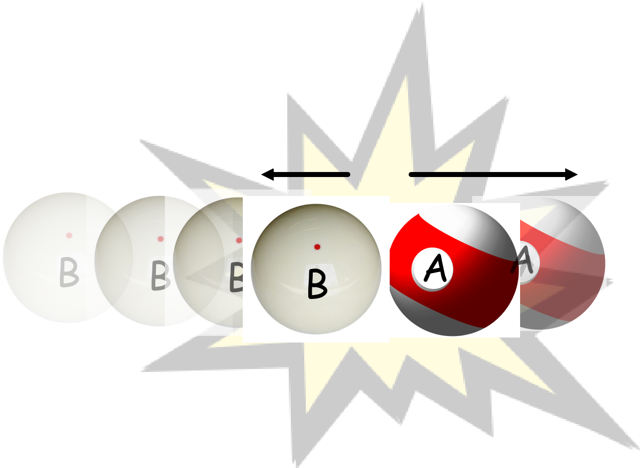
- A **system** can be any specified collection of objects.
- It is **closed** if objects neither enter or leave it
- It is **isolated** if no external force is exerted on it.

Billiards Example



Initially the cue ball (ball B) is moving with some momentum $\vec{p}_B$ and the striped ball (ball A) with momentum $\vec{p}_A$

Initial momentum = $\vec{p}_A + \vec{p}_B$

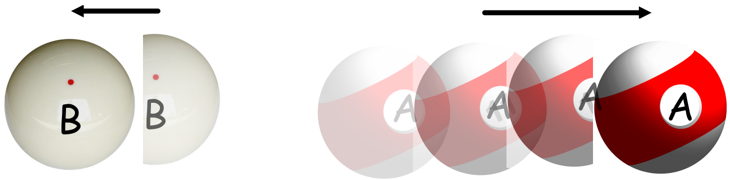


When they collide, ball B exerts a positive force on ball A, and ball A an *equal* force in the opposite direction to ball B. This occurs over some time interval (Δt).

Impulse: $(+\vec{F}_B \Delta t) + (-\vec{F}_A \Delta t) = 0$

This implies that the momentum change is also closed.

Momentum change: $(+\Delta \vec{p}_B) + (-\Delta \vec{p}_A) = 0$



So the new momentums in ball A and ball B are:

$$\vec{p}'_A = \vec{p}_A + \Delta\vec{p}$$

$$\vec{p}'_B = \vec{p}_B + (-\Delta\vec{p})$$

Looking at these equations we can see the net change in momentum is zero.

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

Momentum is Conserved!

So, the Law of Conservation of Momentum states: *The momentum of any closed, isolated system does not change.*

Example:

Two freight cars A and B, each have a mass of 3.0×10^5 kg. Car B is moving at +2.2 m/s while Car A is at rest.

When Car B collides with Car A, they will couple together and move as one unit. We can use the conservation of momentum to find the velocity of the coupled cars.

*Assume in this case the cars roll without friction so there is no net external force. Therefor, the two cars make up a closed, isolated system and momentum is conserved.

Finding the initial momentum...

$$\vec{p}_A + \vec{p}_B$$

$$m\vec{v}_A + m\vec{v}_B$$

$$(3.0 \times 10^5 \text{ kg})(0 \text{ m/s}) + (3.0 \times 10^5 \text{ kg})(+2.2 \text{ m/s})$$

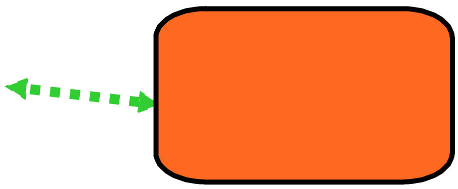
Initial Momentum = 6.6×10^5 kgm/s

After the collision the two coupled cars have the same velocity. With their masses being equal this means their momentum is also the same.

Final momentum

$$\vec{p}'_A + \vec{p}'_B$$

$$m\vec{v}'_A + m\vec{v}'_B$$



$$2m\vec{v}'$$

$$2(3.0 \times 10^5\ \text{kg})\vec{v}'$$

Final momentum = $(6.0 \times 10^5\ \text{kg})\vec{v}'$

By the Law of the Conservation of Momentum

$$\vec{p}_A + \vec{p}_B = p'_A + p'_B$$

$$6.6 \times 10^5 kgm/s = (6.0 \times 10^5 kgm/s)\vec{v}'$$

$$+1.1 \text{ m/s} = v'$$

The final velocity of the Cars moving together is +1.1 m/s

Example 2:

Glider A of mass 0.355 kg moves along a frictionless air track with a velocity of 0.095 m/s. It collides with glider B of mass 0.710 kg moving in the same direction at a speed of 0.045 m/s. After the collision, glider A continues in the same direction with a velocity of 0.035 m/s. What is the velocity of glider B after the collision?

Example 3

An astronaut at rest in space with mass 84 kg fires a thruster that expels 35 g of hot gas at 875 m/s. What is the velocity of the astronaut after firing the shot?

Try Questions 5-11 on pages 185,188-189