SOLVING QUADRATICS
by
Completing the Square

*When equations aren't easily factorable, we may need to complete the square to solve them.

Ex.1: Solve by completing the Square:

$y = x^2 - 6x + 4$

$0 = (x^2 - 6x + 9) + 4 - 9$

$0 = (x-3)^2 - 5$

$5 = (x-3)^2$

$\pm\sqrt{5} = x - 3$

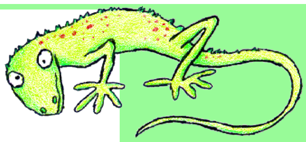
$3 \pm \sqrt{5} = x$

Think About It.

EXACT ROOTS APPROXIMATE ROOTS

So x = $3+\sqrt{5}$ and $3-\sqrt{5}$ (5.24 and 0.76)

Solving Quadratics by Comp the Square



Ex. 2: $y = 2x^2 - 3x - 1$

$$0 = 2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - 1 - \frac{18}{16}$$

$$0 = 2\left(x - \frac{3}{4}\right)^2 - \frac{34}{16}$$

$$\frac{17}{8} = 2\left(x - \frac{3}{4}\right)^2$$

$$\frac{17}{16} = \left(x - \frac{3}{4}\right)^2$$

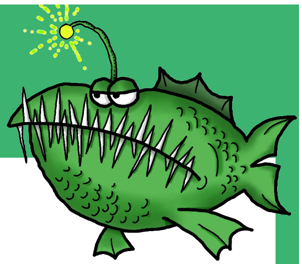
$$\sqrt{\frac{17}{16}} = x - \frac{3}{4}$$

$$\pm\frac{\sqrt{17}}{4} = x - \frac{3}{4}$$

$$\frac{3}{4} \pm \frac{\sqrt{17}}{4} = x$$

$$\boxed{\frac{3 \pm \sqrt{17}}{4} = x}$$

So, $x = 1.78$ and -0.28



Ex.3: $(x + 4)^2 = 9$

$x + 4 = \pm 3$

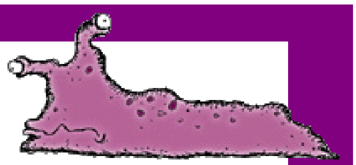
$x = -4 \pm 3$. So, the x-intercepts are **-1 and -7.**

Ex.4: $(x + 1)^2 = 6$

$x + 1 = \pm \sqrt{6}$

$x = -1 \pm \sqrt{6}$. So, the x-intercepts are **1.45 and -3.45**

Solving Quadratics by Comp the Square



Ex.5: $y = 3x^2 - 6x - 1$

$$0 = 3(x^2 - 2x + 1) - 1 - 3$$

$$0 = 3(x - 1)^2 - 4$$

$$4 = 3(x - 1)^2$$

$$\frac{4}{3} = (x - 1)^2$$

$$\sqrt{\frac{4}{3}} = x - 1$$

$$\pm\frac{2}{\sqrt{3}} = x - 1$$

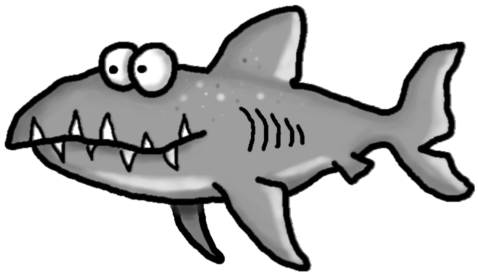
$1 \pm \frac{2}{\sqrt{3}} = x$. We cannot leave a radical as a denominator!

$1 \pm \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ So, rationalize by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ or 1.

$\boxed{1 \pm \frac{2\sqrt{3}}{3} = x}$ So, the x ints are 2.16 and - 0.16

Solving Quadratics by Comp the Square

Ex.6: $(x+5)^2 = \frac{9}{8}$



$x + 5 = \pm\sqrt{\frac{9}{8}}$

$x + 5 = \pm\frac{3}{\sqrt{8}}$

$x = -5 \pm\frac{3}{\sqrt{8}}$ (cannot leave a radical on bottom)

$x = -5 \pm\frac{3}{\sqrt{8}}\cdot\frac{\sqrt{8}}{\sqrt{8}}$

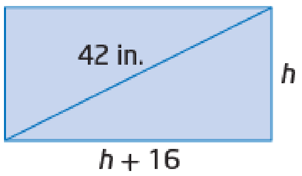
$x = -5 \pm\frac{3\sqrt{8}}{8}$ (reduce $3\sqrt{8}$ to $3\sqrt{4\cdot 2}$ or $3\cdot 2\sqrt{2}$ which is

$x = -5 \pm\frac{6\sqrt{2}}{8}$

$x = -5 \pm\frac{3\sqrt{2}}{4}$ So, the x-ints are -3.94 and -6.06

<u>Ex. 7:</u> A wide-screen t.v. has a diagonal measurement of 42 inches. The width of the screen is 16 inches more than the height. Determine the dimensions of the screen to the nearest tenth of an inch.

Draw a diagram. Let h represent the height of the screen. Then, $h + 16$ represents the width of the screen.



Use the Pythagorean Theorem.

$$
\begin{aligned}
h^2 + (h + 16)^2 &= 42^2 \\
h^2 + (h^2 + 32h + 256) &= 1764 \\
2h^2 + 32h + 256 &= 1764 \\
2h^2 + 32h &= 1508 \\
h^2 + 16h &= 754 \\
h^2 + 16h + 64 &= 754 + 64 \\
(h + 8)^2 &= 818 \\
h + 8 &= \pm\sqrt{818} \\
h &= -8 \pm \sqrt{818}
\end{aligned}
$$

$h = -8 + \sqrt{818}$ or $h = -8 - \sqrt{818}$

$h \approx 20.6$ $\quad$ $h \approx -36.6$

Since the height of the screen cannot be negative, $h = -36.6$ is an **extraneous root**.

Thus, the height of the screen is approximately 20.6 in., and the width of the screen is approximately 20.6 + 16 or 36.6 in..

Hence, the dimensions of a 42-in. television are approximately 20.6 in. by 36.6 in..

Check:
$20.6^2 + 36.6^2$ is 1763.92, and $\sqrt{1763.92}$ is approximately 42, the diagonal of the television, in inches.

<u>Ex. 8:</u> Solve $x^2 - 21 = -10x$ by completing the square. Express your answers to the nearest tenth.

<u>Ex. 9:</u> Determine the roots of $-2x^2 - 3x + 7 = 0$ to the nearest hundredth. Then, using technology, verify your answers.